Abstract—This paper presents a force and moment blending control scheme for a dual-controlled missile with tail fins and reaction jets, which is especially aimed at a fast response. A dual missile can be controlled by forces and moments independently, when the reaction jet is located in front of the center of gravity of the missile. Controlling the net force of aerodynamic lift and jet thrust yields much a faster response compared with controlling the net moment, but it uses large control efforts caused by divergence of angle of attack, especially the jet thrust. Here, force control is implemented by using sliding mode control. Large input usage is alleviated by shaping angle of attack. The proposed blending scheme begins with force control and then makes a transition to moment control. Both control strategies are demonstrated by a nonlinear missile system. The smooth transition from force control to moment control is demonstrated. The proposed approach shows a very fast response, while its input usage is almost same as the conventional moment control.

I. INTRODUCTION

Tracking response, rather than maneuver capability, is a major factor that determines the miss-distance of a missile for high altitude interception [1]. Thus fast responses are required for anti-air missiles that engage with highly maneuverable targets. However, it is usually difficult to achieve a sufficiently fast response if the missile is controlled only by aerodynamic fins, since effectiveness of tail fins depends highly on the dynamic pressure especially at high altitude. Moreover, tail-fin-only missiles have a non-minimum phase characteristics which causes a large undershoot [2].

Agile missiles usually combine reaction jets with aerodynamic fins to satisfy requirements for fast responses [3], [4], [5]. Such missiles are usually referred as dual missiles. The agility of the dual missiles is increased due to the additional forces and moments supplied by the reaction jet [1], [2]. However, as explained in [6], a combination of individual solutions from tail fin and reaction jet control does not provide satisfactory performance. Therefore, a systematic approach that considers the effective cooperation between tail fins and reaction jets is necessary [7], [8].

A dual missile can be controlled by either forces or moments when the reaction jet is located in front of the center of gravity of the missile [9]. Previous works on dual missiles, such as [10], have focused on controlling the moments on the missile in order to achieve a desired acceleration by the aerodynamic lift after the angle of attack has been generated as shown in Fig. 1(a). This control scheme is referred as moment control [9]. On the other hand, dual missiles can also be controlled by pure force. In this scheme, control inputs are used to generate force directly to track the acceleration command as shown in Fig. 1(b). This control scheme is referred as force control [9].

In moment control, aerodynamic force and jet thrust cooperate to generate a desired moment. However, the direction of the aerodynamic force induced by the tail fin is in the opposite direction to the desired acceleration as shown in Fig. 1(b). This reduces the efficiency of control and may also result in non-minimum phase. Moreover, the slew-rate of the tail fin is large in moment control, which constrains the response speed of the missile. On the other hand, in force control, it has been shown that the transfer function from force to acceleration is nearly a constant [9]. This implies that force inputs translate directly to an acceleration. Thus, force control reduces the slew-rate of the tail fin deflection up to half when compared with the moment control case [11]. However, the angle of attack is not controlled anymore because acceleration is maintained from actuator forces directly rather than from the lift force generated from the angle of attack. Unfortunately, the uncontrolled angle of attack diverges which causes large input usage, and this phenomenon constrains the utility of the force control.

In short, force control is a desired solution when the tracking response is critical and the control effort is not a concern. Furthermore, a blended control scheme of the two, i.e. , moment control and force control, that starts out as force control and makes a transition to moment control can reduce control effort while maintaining a sufficiently fast response in tracking the acceleration command. However, blending the two control schemes is a non-trivial task since a change in the direction of the tail fin is required [11], it may destabilize the missile system during transition. Moreover, it is not simple to choose a criterion to activate transition between the force and moment control in a nonlinear setting.

In this paper, we realize a force control scheme on a dual missile, which uses smaller control inputs than conventional force control. Also, we propose a blending scheme that makes a transition from force control to moment control in a efficient manner. The control allocation between the tail...
fin and reaction jet is achieved by generating an additional moment command in both schemes. The additional moment command is derived such that the time response of the angle of attack can be adequately shaped. In the force control case, the moment command is generated to make the angle of attack converge to zero, and in the blending control scheme, the moment command is generated to shape the angle of attack response so that it converges to that of the moment control case. We refer this control scheme as transition control. For convenience of analysis and design, the reaction jet is treated as a continuous variable as in [5] and [12].

The rest of the paper is organized as follows. In Section II, the dual missile model is presented. In Section III, force control is developed using sliding mode control, and blending scheme for transition from force control to moment control is developed. In Section IV, the nonlinear dynamic system of a dual missile is numerically simulated to verify the performance of proposed controllers. Finally, concluding remarks are given in Section V.

II. MISSILE MODEL

The missile configuration considered in this paper includes both tail fins and reaction jets. Both types of control effectors produce redundant moments and forces on the missile. Short-period approximation of the longitudinal dynamics for a dual missile is as follows:

\[
\dot{\alpha} = q + \frac{\cos \alpha}{m V_T} QS(C_{z\alpha}' + C_{z\delta} \dot{\delta}) + \frac{\cos \alpha}{m V_T} T_z
\]

\[
\dot{q} = \frac{QSD}{I_{yy}} (C_{m\alpha}' + C_{m\delta} \frac{D}{2V_T} q + C_{m\delta} \dot{\delta}) - \frac{l_i}{I_{yy}} T_z
\]

\[
A_z = \frac{1}{m} QS(C_{z\alpha}' + C_{z\delta} \dot{\delta}) + \frac{1}{m} T_z,
\]

where \( \alpha \), \( q \), \( A_z \) are angle of attack, pitch rate, \( z \)-direction acceleration, and \( V_T \), \( m \), \( Q \), \( S \) and \( D \) are total missile velocity, total missile mass, dynamic pressure, reference area and reference length, respectively. \( C_{z\alpha}' \) and \( C_{m\alpha}' \) are the pitch force coefficient and the pitch moment coefficient due to angle of attack, and \( C_{m\delta} \) is the pitch moment coefficient due to the pitch rate. \( C_{z\delta} \) and \( C_{m\delta} \) are the pitch force coefficient and the pitch moment coefficient due to the fin deflection. \( I_{yy} \), \( l_i \) are the pitch moment of inertia and the reaction jet distance from center of gravity. \( \delta_q \) and \( T_z \) are the fin deflection and the reaction jet thrust.

In the longitudinal dynamics, \( z \)-direction force and pitch moment dynamics can be described by

\[
F_z = QS(C_{z\alpha}' + C_{z\delta} \dot{\delta}) + T_z
\]

\[
M = QSD(C_{m\alpha}' + C_{m\delta} \frac{D}{2V_T} q + C_{m\delta} \dot{\delta}) - l_i T_z.
\]

In the process of designing the controller, differentiations of \( C_{z\alpha}' \) and \( C_{m\alpha}' \) are used. We express these aerodynamic coefficients using a least square curve fitting as

\[
C_{z\alpha}' \approx p_1 \alpha + p_2
\]

\[
C_{m\alpha}' \approx p_3 \alpha + p_4.
\]

III. CONTROLLER DESIGN

The requirement for control allocation arises from the capability of using more than one set of control effectors on a dual missile [9]. Thus control allocation determines the unique solution from various solution sets to tack a given acceleration command as shown in Fig. 2(a).

In this paper, we derive a desired moment in addition to given acceleration command to determine the unique solution as the dot line \( M_d \), shown in Fig. 2(b) from the command generator block. This implies that the derived desired moment determines blending schemes of tail fins and reaction jets while tracking the acceleration command. Thus defining \( M_d \) is the most important task in our approach. We define desired moment to implement force control, and we also improve the controller in terms of control usages in Section III-A. Next, we derive a desired moment which can generate smooth transition from force control to moment.
control in Section III-B. This allows to shape the angle of attack response as wanted. Finally, stability analysis is described in Section III-C.

A. Force Control

In this section, we design a sliding mode nonlinear controller to implement force control for a fast response. To track the acceleration command, the error is defined as

\[ e_1 = A_z - A_{zd} , \tag{10} \]

where \( A_{zd} \) is a desired acceleration. In our approach, we have to derive an additional command to determine blending scheme of control inputs as shown in Fig. 2(b). The main feature of the force control is that moments produced by tail fins and reaction jets cancel each other out. Thus in order to design the force controller, the additional command should express that feature. In this point of view, the additional command is determined as desired moment, and desired moment \( M_d = 0 \) at this time. After generating the command, the control law is derived from sliding mode control. Sliding surfaces are defined as

\[ s_1 = e_1 + \lambda \int e_1 \, dt \tag{11} \]
\[ s_2 = M - M_d , \tag{12} \]

where \( \lambda \in R \) is a positive constant. Here, \( s_1 \) is defined to track the acceleration command and \( s_2 \) is defined to track the derived moment, respectively. Unless we define the \( s_2 \), control inputs cannot be determined from various solution sets, thus the control allocation problem is blended in the control design problem. When system states are in the sliding mode, moment is enforced to be zero. This implies that control inputs are used to generate direct forces to achieve the acceleration command. Thus force control can reduce the slew-rate of tail fin deflection effectively.

However angle of attack is not controlled in force control, because moment is always enforced to zero regardless of angle of attack. From (1), (2), (3), (5) and by assuming \( \alpha \) is a small value, angle of attack and pitch angle dynamics are expressed as

\[ \dot{\alpha} = q + \frac{A_z}{V_T} , \tag{13} \]
\[ \dot{q} = \frac{M}{I_{yy}} . \tag{14} \]

From (13), \( \alpha \) increases or decreases continuously depending on the acceleration command unless \( q \) is controlled. Unfortunately, the angle of attack diverges in a harmful direction, i.e. when the acceleration is maintained by the angle of attack, positive acceleration needs negative angle of attack. Thus, even though force control can have a very fast response, it may use large control inputs.

In order to prevent angle of attack from diverging, we feedback the angle of attack from the missile dynamics to the command generator block as shown dashed line in Fig. 2(a).

From this, we put a relationship between the angle of attack and the desired moment as

\[ M_d = -K_p\alpha - K_d\dot{\alpha} , \tag{15} \]

where \( K_p \) and \( K_d \) are positive constant values. From this relationship, moment is not always enforced to zero. At this time, when \( \alpha \) increases or decreases, the desired moment has some value to push \( \alpha \) to zero. If \( A_z \) follows \( A_{zd} \), and \( M \) follows \( M_d \), then (13) and (14) with (15) give

\[ \dot{\alpha} = q + \frac{A_{zd}}{V_T} , \tag{16} \]
\[ \dot{q} = -K_p\alpha - K_d\dot{\alpha} . \tag{17} \]

Integrating (17) and substituting into (16) yields

\[ \dot{\alpha} = \int -K_p\alpha - K_d\dot{\alpha} + \frac{d}{dt} \left( \frac{A_{zd}}{V_T} \right) , \tag{18} \]

and differentiating (18) yields

\[ \ddot{\alpha} = -K_p\alpha - K_d\dot{\alpha} + \frac{d}{dt} \left( \frac{A_{zd}}{V_T} \right) . \tag{19} \]

In practice, since the total speed of the missile is very fast, the last term in the right-hand side of (19) is negligible. We can rewrite (19) with respect to angle of attack as

\[ \ddot{\alpha} + K_p\alpha + K_d\dot{\alpha} = 0 , \tag{20} \]

where

\[ K_p = \frac{K_p}{I_{yy}} , \quad K_d = \frac{K_d}{I_{yy}} . \]

(20) implies that the desired moment (15) makes the angle of attack converge to zero at steady state, when we choose \( K_p \) and \( K_d \) properly. After \( \alpha \) approaches to zero, the moment also approaches to zero. Therefore this also functions like a force controller.

The control problem is to asymptotically drive the missile states to track the desired commands. Defining sliding surfaces same as (11), (12) and substituting (15) yields

\[ s_1 = e_1 + \lambda \int e_1 \, dt \]
\[ s_2 = M + K_p\alpha + K_d\dot{\alpha} . \tag{21} \]

When the system states are in the sliding mode, the acceleration command and desired moment are achieved. Now the angle of attack is enforced to be zero during the force control mode. In this way, wasting of input deflections caused by divergence of angle of attack can be reduced.

The existence of a sliding mode, \( s_i = 0 \), requires \( \dot{s}_i = 0 \):

\[ s_1 = e_1 + \lambda e_1 = A_z - A_{zd} + \lambda e_1 = 0 \tag{22} \]
\[ s_2 = M + A_{zd} + \lambda e_1 = 0 . \tag{23} \]

In order to find a control law which makes \( \dot{s}_i = 0 \), we need \( A_z, M \) and \( \dot{\alpha} \). For simplicity of derivation, we simplify the actuators as

\[ \dot{\delta}_q = \frac{\delta_{qc} - \delta_q}{\tau_q} , \quad \dot{T}_z = \frac{T_{zc} - T_z}{\tau_z} , \tag{24} \]
where $\tau_q$ is the time constant of tail fins. We can obtain $A_z, M$ and $\dot{\alpha}$ by differentiating (1), (3) and (5) as follows:

$$A_z = \frac{1}{m} Q S(C_i'z + C_z \dot{\delta}_q) + \frac{1}{m} \dot{T}_z + d_{A_z}$$

$$= \Psi_1 + b_{11} \delta_q + b_{12} T_z + d_{A_z} \tag{25}$$

$$\dot{M} = Q S D \left( C_m' + C_m \frac{D}{2V_T} \dot{q} + C_m \dot{\delta}_q \right) - l_t \dot{T}_z + d_M$$

$$= \Psi_2 + b'_{21} \delta_q + b'_{22} T_z + d_M \tag{26}$$

$$\dot{\alpha} = \dot{q} - \sin \alpha A_z \dot{\alpha} + \cos \alpha A_z$$

$$= \dot{q} - \sin \alpha V_T A_z \dot{\alpha} + \cos \alpha V_T (\Psi_1 + b_{11} \delta_q + b_{12} T_z) \tag{27}$$

where

$$d_{A_z} = \frac{1}{m} Q S C_{z_{s}} \dot{\delta}_q \tag{28}$$

$$d_M = Q S D \left( C_m' + C_m \frac{D}{2V_T} q + C_m \frac{d}{dt} \left( \frac{D}{2V_T} \right) q + C_m \dot{\delta}_q \right) \tag{29}$$

$$\Psi_1 = \frac{1}{m} Q S \left( p_1 \dot{\alpha} - \frac{C_z \dot{\delta}_q}{\tau_q} \right) - \frac{T_z}{m \tau_z} \tag{30}$$

$$b_{11} = \frac{Q S C_{z_{s}}}{\tau_q}, \quad b_{12} = \frac{1}{m \tau_z} \tag{31}$$

$$\Psi_2 = Q S D \left( p_2 \dot{\alpha} + C_m \frac{D}{2V_T} \dot{q} + \frac{C_m \delta_q}{\tau_q} \right) + \frac{l_t}{T_z} \tag{32}$$

$$b'_{21} = \frac{Q S D C_{m} \dot{\delta}_q}{\tau_q}, \quad b'_{22} = -\frac{l_t}{\tau_z} \tag{33}$$

$d_{A_z}$ and $d_M$ can be viewed as disturbance terms. Substituting (25), (26) and (27) into (22) and (23) yields

$$\delta_1 = v_1 + b_{11} \delta_q + b_{12} T_z \tag{34}$$

$$\dot{s}_1 = \nu_1 + b_{11} \delta_q + b_{12} T_z \tag{35}$$

where

$$\nu_1 = \Psi_1 - A_z \dot{s}_d + \lambda e_1 \tag{36}$$

$$\nu_2 = \Psi_2 + K_p \dot{\alpha} + K_d \left( \dot{q} - \sin \alpha A_z \dot{\alpha} + \cos \alpha V_T \Psi_1 \right) \tag{37}$$

$$b_{21} = K_d \cos \alpha V_T b_{11} + b'_{21} \tag{38}$$

$$b_{22} = K_d \cos \alpha V_T b_{12} + b'_{22} \tag{39}$$

In matrix form, we have

$$\dot{s} = \nu + Bu \tag{40}$$

where

$$s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}^T, \quad \nu = \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix}^T$$

$$u = \begin{bmatrix} \delta_q \\ T_z \end{bmatrix}^T, \quad B = \begin{bmatrix} b_{11} \\ b_{12} \end{bmatrix}^T \tag{41}$$

Thus the equivalent control input $\dot{u}$ is defined as

$$\dot{u} = -B^{-1} \nu \tag{42}$$

and the control input is

$$u = B^{-1} (-\nu - k_1 s - k_2 \text{sgn}(s)) \tag{43}$$

Since $\tau_q > 0, \tau_z > 0$ and $l_t > 0$, from (31), (38) and (39)

$$|B| = \begin{vmatrix} b_{11} \\ b_{21} \\ b_{22} \end{vmatrix} \neq 0. \tag{44}$$

Therefore $\delta_q$ and $T_z$ can be uniquely solved by (43).

B. Transition Control

In this section, we design transition control which start from force control and then makes a transition to moment control. Even though force control is improved by preventing divergence of angle of attack, it uses larger inputs than the moment control. Furthermore, there exists limit of tail fin to get a direct lift response in the low dynamic pressure. Thus for fast response during large guidance command, the missile should be start from force control and then make a transition to moment control.

In order to design transition control, we introduce new variable $q_d$ from the moment control which can be defined from $A_z d$ [10] as described in the following paragraph.

When the desired acceleration is maintained by the aerodynamic lift after angle of attack has been generated, then $\delta_q = 0, T_z = 0$ in (3) as

$$A_{zd} = \frac{1}{m} Q S C_{z_{d}} \tag{45}$$

Substituting the approximation of $C_{z_{d}}$ in (6) and differentiating (45) yields

$$A_{zd} = \frac{1}{m} Q S (p_1 \dot{\alpha}_d) \tag{46}$$

rearrange (46) with respect to $\alpha_d$ as

$$\alpha_d = \frac{m}{Q S p_1} A_{zd} \tag{47}$$

When $\dot{\alpha}$ follows $\dot{\alpha}_d$ and $q$ follows $q_d$, then (13) is as

$$\alpha_d = q_d + \frac{1}{V_T} A_{zd} \tag{48}$$

Substituting (47) into (48) and rearranging on $q_d$ yields

$$q_d = \frac{m}{Q S p_1} A_{zd} - \frac{1}{V_T} A_{zd} \tag{49}$$

Now we define new error as

$$e_2 = q - q_d \tag{50}$$

Because $q_d$ is derived from assuming that acceleration is maintained by the aerodynamic lift from angle of attack, this additional command makes dual controller tend to the moment control. But in our approach, we use an additional desired moment to make blending scheme of a dual missile.

Here, we derive a advanced desired moment which can implement the transition condition as

$$M_d = -K_p \alpha - K_d \dot{\alpha} - K_I \int e_2 \, dt \tag{51}$$

where $K_p, K_d$ and $K_I$ are positive constant values.
Let us now define the control input
\[ u = B^{-1}(\nu - K_I e_2 - k_1 s - k_2 sgn(s)) . \] (64)
C. Stability Analysis

In this section, the Lyapunov theorem is employed to prove the tracking stability of the closed-loop system under the transition control. Let us consider the positive definite Lyapunov function

\[ V = \frac{1}{2} s^T s. \]  

(65)

In order to move the system states toward the sliding surfaces, the time derivative of the function must be negative definite. Take the time derivative of (65) and substitute (62) and (64)

\[ V' = s^T \dot{s} \]
\[ = s^T (-k_1 s - k_2 \text{sgn}(s)) \]
\[ = -k_1 s^T s - k_2 \|s\|_1 \leq 0. \]

Thus, when control input (64) is employed, the tracking error asymptotically converges to zero, and angle of attack is also shaped as (56).

IV. NUMERICAL SIMULATION

In this section, numerical results of the proposed transition control are reported. A second-order filter is used to shape acceleration command as

\[ A_{zd} = \frac{s^2 + 2\zeta_{cmd}\omega_{cmd}s + \omega_{cmd}^2}{s^2 + 2\zeta_{cmd}\omega_{cmd}s + \omega_{cmd}^2} A_{zd}. \]  

(66)

where \( \omega_{cmd} \) is a natural frequency and \( \zeta_{cmd} \) is a damping ratio of the filter. The nonlinear model is considered and an aerodynamic coefficient database constructed experimentally was used. The parameters of the missile model are presented in TABLE II.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>( D )</td>
<td>Reference length</td>
<td>0.3 m</td>
</tr>
<tr>
<td>( S )</td>
<td>Reference area</td>
<td>0.707 m²</td>
</tr>
<tr>
<td>( m )</td>
<td>Total missile mass</td>
<td>200 kg</td>
</tr>
<tr>
<td>( I_{yy} )</td>
<td>Pitch moment of inertia</td>
<td>450 kgm²</td>
</tr>
<tr>
<td>( l_z )</td>
<td>Reaction jet distance from center of gravity</td>
<td>1.2 m</td>
</tr>
<tr>
<td>( \omega_n )</td>
<td>Natural frequency of the tail fin</td>
<td>60 Hz</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>Damping ratio of the tail fin</td>
<td>0.7</td>
</tr>
<tr>
<td>( \tau_z )</td>
<td>Time constant of reaction jet</td>
<td>0.001 s</td>
</tr>
</tbody>
</table>

The two-step acceleration command, filtered with \( \omega_{cmd} = 80 \) Hz and \( \zeta_{cmd} = 0.707 \), is simulated to demonstrate fast response of proposed transition control, and the results are shown in Fig. 3. Here, we show only the moment control and the transition control to compare the response. In Fig. 3(a), we see that the transition control can follow the fast guidance command, while the moment control cannot follow the command owing to requirement for fast response. The angle of attack in the transition control follows the angle of attack in the moment control as shown in Fig. 3(b). Fig. 3(c) and (d) show the time history of the input commands.
The results of simulation in the case of a 10g guidance command are shown in Fig. 4. Here, acceleration command is filtered with $\omega_{cmd} = 60 \, Hz$ and $\xi_{cmd} = 0.707$ for demonstrating transition from force control to moment control. We can see that force control has a fast settling time than the moment control. In Fig. 4(d), we can see that moment control uses smaller reaction jet input than that of force control. Also in Fig. 4(c), the initial direction of tail fin deflection in the force control is opposite to the that of moment control, because the tail fin is used to generate lift force directly in the force control. We can see in Fig. 4(c) and (d) that transition control initially starts from force control and changes the trend toward moment control smoothly. This smooth transition is done by shaping the angle of attack like that of moment control, thus the angle of attack in transition control follows that of moment control as shown in Fig. 4(b). The transition speed can be controlled by changing the values of $K_p$, $K_d$ and $K_I$.

V. CONCLUSIONS

In this paper, blending schemes for a dual-controlled missile with tail fins and reaction jets are developed, especially to achieve a fast response. The force control has a very fast response at beginning, but it uses large inputs to maintain the acceleration. On the other hand, moment control has a relatively slow response but it uses small inputs. Thus we developed a blending scheme which can start from force control and then makes a transition to moment control. The problem is how to realize cooperation of tail fin and reaction jet for this transition control while following the acceleration command. In this work, we derive an additional command to determine unique solution. We choose an additional command as a desired moment. We find that when dual missile is controlled by pure forces, the angle of attack diverges. Thus we use the feedback of angle of attack and consider it to determine the desired moment. First, we design a force controller with the desired moment which can enforce angle of attack to maintain zero. The designed force controller reduces wasting of input usages by preventing the divergence of angle of attack. Second, the desired moment is revised to realize transition control which starts from force control and then makes a transition to moment control. The key is shaping the angle of attack. We put a relationship between the angle of attack and desired moment so that the angle of attack is shaped like that of moment control. The transition controller designed in this research starts from the force control and then makes a smooth transition to moment control. The smooth transition from force control to moment control is demonstrated by simulation with nonlinear missile system. The proposed transition controller has a very fast response, while its input usage is almost same as the conventional moment control.

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