Robust Proportional Navigation Guidance Against Highly Maneuvering Targets
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Abstract: A robust proportional navigation guidance law is proposed by using Lyapunov approach. Proportional navigation guidance(PNG) law is popularly adopted for missile guidance because it is powerful and simple to implement. In fact, it has been shown that PNG is optimal solution for a stationary target. But PNG performance may sharply degrade in the presence of highly maneuvering target. To find estimation of the miss distance, an optimal approach is used. Then switching control is derived by using this estimation of miss distance. The proposed robust guidance has the form of combination of the conventional PNG and the switching part. Several nonlinear engagement simulations show that the proposed guidance law is robust to the highly maneuvering target.

Keywords: Guidance Law, Robustness, Miss distance

1. INTRODUCTION

Proportional navigation guidance(PNG) is popularly adopted for missile guidance because it is powerful and simple to implement. In fact, it has been shown that PNG is the optimal solution for linearized engagement equations with constant speed of the missile and stationary target [1]. But PNG performance may sharply degrade in the presence of highly maneuvering targets. Augmented proportional navigation(APN) is a modified PNG taking into account target maneuver. However, when the target acceleration is unknown or is poorly estimated, it cannot be implemented in practice.

In order to overcome the above problem, some robust control approaches, such as \( H_\infty \) control and sliding mode control, have been used to derive guidance law [2], [3], [4], [5]. Yang et al. proposed \( H_\infty \) robust guidance law with nonlinear kinematics by solving the Hamilton-Jacobi partial differential inequality(HJPDI). But it is not a simple task to find an analytic solution to the HJPDI. Since sliding mode control is robust to uncertainties and disturbance, it has been applied to many guidance problems. Babu et al. studied the guidance law for highly maneuvering targets using the sliding surface of the zero line-of-sight(LOS) rate based on Lyapunov method. Zhou et al. proposed an adaptive sliding mode guidance law using linearized equations. Jongki et al. studied the missile guidance by variable structure control. In the missile guidance, LOS rate is a main factor to catch the target. For that reason sliding surface is usually defined as a LOS angular rate to nullify targets maneuvering.

In this paper, we derive robust guidance law based on the conventional PNG to maintain its optimal properties. In order to find the performance criterion of a guidance law, we first derive estimation of miss distance for the nonmaneuvering target. And then, check the stability of this estimation value when the target acceleration exists, which is unknown disturbance. By using Lyapunov approach to find a switching control part to nullify target’s maneuvering, zero miss distance is guaranteed. Linearized equations are used to derive estimation of miss distance and switching control for simplicity.

The rest of this paper organized as follows. In Section 2, two-dimensional engagement is presented by using nonlinear differential equations. Also linearized equation of motions are derived to design guidance law. In Section 3, estimation of miss distance is derived for the nonmaneuvering target. After that, Lyapunov approach is used to complete the guidance law for the maneuvering target. In Section 4, nonlinear engagement simulation is conducted to show performance of proposed guidance law. Finally, concluding remarks are given in Section 5.

2. ENGAGEMENT GEOMETRY

In this section, the missile target relative kinematics is presented. Two-dimensional engagement model is considered as shown in Fig. 1. Here, we treat missile and target as point mass model and assume that missile and target velocities are constants. Then, engagement model...
can be represent the following differential equations:

\[
\begin{align*}
\dot{y} &= V_T \sin \gamma_T - V_M \sin \gamma_M \\
&\approx V_T \gamma_T - V_M \gamma_M \\
V_T \gamma_T &= a_T \\
V_M \gamma_M &= a_M
\end{align*}
\]

where \( y \) is the \( y \)-axis relative distance between the missile and the target, \( \gamma_M \) and \( \gamma_T \) are the flight path angles of the missile and the target, and \( V_M, V_T \) are the velocities of the missile and the target. By assuming that \( \gamma_M \) and \( \gamma_T \) are small then the linearized equation of motion can be written as

\[
\begin{align*}
\dot{\xi}_1 &= \xi_2 - \xi_3 \\
\dot{\xi}_2 &= a_T \\
\dot{\xi}_3 &= a_M
\end{align*}
\]

where

\[
\begin{align*}
\xi_1 &= y, \quad \xi_2 = V_T \gamma_T, \quad \xi_3 = V_M \gamma_M
\end{align*}
\]

### 3. GUIDANCE LAW

In this section, we derive estimation of the miss distance when the optimal control input is used with a stationary target \( (a_T = 0) \). After that, we define Lyapunov function as a function of the estimated miss distance to design robust guidance when the target acceleration exists.

#### 3.1 PNG: Optimal approach

The quadratic cost function for the finite time problem can be expressed as

\[
J = \frac{\rho}{2} \xi_1(t_f)^2 + \frac{1}{2} \int_{t_0}^{t_f} (a_M(s))^2 \, ds
\]

where the subscript \( f \) denotes the state variable at homing and \( \rho \) is a weight parameter for the miss distance. In order to solve this optimization against a stationary target, the Hamiltonian can be written as

\[
H = \frac{1}{2} a_M^2 + \lambda_1 (\xi_2 - \xi_3) + \lambda_3 a_M
\]

The adjoint equations are given by

\[
\begin{align*}
\dot{\lambda}_1 &= 0, \quad \lambda_1(t_f) = \rho \xi_1(t_f) \\
\dot{\lambda}_2 &= -\lambda_1, \quad \lambda_2(t_f) = 0 \\
\dot{\lambda}_3 &= \lambda_1, \quad \lambda_3(t_f) = 0
\end{align*}
\]

Integrating (8) yields the following solutions

\[
\begin{align*}
\lambda_1 &= \rho \xi_1(t_f) \\
\lambda_2 &= \rho \xi_1(t_f) t_{go} \\
\lambda_3 &= -\rho \xi_1(t_f) t_{go}
\end{align*}
\]

where \( t_{go} = t_f - t \). From the first-order necessary condition, the optimal solution of the missile acceleration is

\[
a_M^* = -\lambda_3 = \rho \xi_1(t_f) t_{go}
\]

Since (12) is a function of the final state variable, \( \xi_1(t_f) \) needs to be expressed in terms of the current state variables. Substituting (12) into the state equation (5) with \( a_T = 0 \) and integrating yields

\[
\xi_1(t_f) = \frac{\xi_1 + \xi_1 t_{go}}{1 + (\rho/3) t_{go}^2} \approx \xi_1 + \xi_1 t_{go}
\]

If the weight parameter \( \rho \) is large enough, \( \xi_1(t_f) \) can be expressed as the final form of (13).

For simple analysis and understanding, it is convenient to use a linearized model [6]. The expression for the line-of-sight (LOS) angle can be linearized using the small-angle approximation as

\[
\lambda = \frac{y}{r}
\]

where \( r \) is the relative distance between the missile and the target. We can also linearize the range equation as

\[
t_{go} = -\frac{r}{\dot{r}}
\]

Differentiating (14) with (15) yields

\[
\dot{\lambda} = \frac{\xi_1 + \xi_1 t_{go}}{V_c(t_{go})^2}
\]

Then \( \xi_1(t_f) \) and \( a_M \) can be expressed by \( \dot{\lambda} \) which can be directly achieved by the seeker as

\[
\xi_1(t_f) = -\frac{3\dot{r}}{\rho t_{go}} \lambda
\]

\[
a_M = -3\dot{r} \lambda = -N \dot{r} \lambda
\]

The optimal solution is the form as PNG when effective navigation constant \( N \) is 3. As shown in the above procedure, PNG is the optimal solution when \( a_T = 0 \). Thus we will analyze estimation of the miss distance \( \xi_1(t_f) \) when the target acceleration is not zero in the next section.

#### 3.2 Robust PNG

In a linearized analysis we will treat the closing velocity as a positive constant as

\[
V_c = -\dot{r} = V_M + V_T \quad \text{(head-on case)}
\]

\[
V_c = -\dot{r} = V_M - V_T \quad \text{(tail chase case)}
\]

In this paper, we assume that \( V_M, V_T \) are constants. Thus we approximates \( \dot{r} \approx 0 \). Target acceleration \( a_T \) is assumed to be bounded as

\[
|a_T| < W
\]
In order to analyze the miss distance when \( a_T \neq 0 \), we define a Lyapunov function as

\[
V = \frac{1}{2} \xi(t_f)^2
\]

Differentiating \( V \) with respect to time yields

\[
\dot{V} = \xi(t_f) \xi(t_f) + \frac{3}{\rho} \left( \frac{\dot{\lambda}^2}{\rho} \right) \left( \frac{3\dot{\lambda}^2 \dot{\lambda} - \dot{\lambda}^3}{\rho} \right)
\]

\[
= \frac{3}{\rho} \left( \frac{\dot{\lambda}}{\rho} \right)^3 \left( -3(\dot{\lambda})^2 - \dot{\lambda} a_M + \dot{\lambda} a_T \right)
\]

\[
= K(t) \left( 3(\dot{\lambda})^2 + \dot{\lambda} a_M - \dot{\lambda} a_T \right)
\]

(23)

where

\[
\dot{\lambda} = -2\dot{\lambda} - a_M + a_T
\]

(24)

\[
K(t) = \frac{3}{\rho} \left( \frac{\dot{\lambda}}{\rho} \right)^3 \left( \frac{3}{\rho} \right)^2 \frac{1}{t_{go}} \geq 0
\]

(25)

Substituting the optimal guidance law into (23) induces

\[
\dot{V} = -K(t)\dot{\lambda} a_T
\]

(26)

When weight parameter \( \rho \) is large enough, then \( \xi(t_f) = 0 \) at first. Thus optimal missile guidance law makes \( \dot{V} = 0 \) zero when the target acceleration is zero. But when the target maneuver exists, the miss distance stability is not guaranteed as shown in (26).

In order to handle unknown target acceleration, we augment optimal guidance input with a switching part as

\[
a_M = -3\dot{\lambda} - Q\text{sign}(\dot{\lambda})
\]

(27)

where \( Q \) is the switching gain defined as \( Q = W + \varepsilon \). Here \( \varepsilon \) is the small positive constant. This proposed guidance consists of the optimal control part when \( a_T = 0 \) and switching control part. Again substituting the proposed guidance law (27) into (23) yields

\[
\dot{V} = K(t) \left( 3(\dot{\lambda})^2 + \dot{\lambda}(-3\dot{\lambda} - Q\text{sign}(\dot{\lambda})) - \dot{\lambda} a_T \right)
\]

\[
= K(t) \left( -Q\dot{\lambda}\text{sign}(\dot{\lambda}) - \dot{\lambda} a_T \right)
\]

(28)

\[
\leq K(t) \left( -Q|\dot{\lambda}| + |\dot{\lambda}|a_T \right)
\]

(29)

\[
\leq K(t) \left( -Q|\dot{\lambda}| + |\dot{\lambda}|W \right)
\]

(30)

\[
= -K(t)|\dot{\lambda}| \varepsilon \leq 0
\]

(31)

which indicates that \( \xi(t_f) \rightarrow 0 \) even though the target acceleration exists.

4. SIMULATION RESULTS

The proposed guidance law is applied to a two-dimensional nonlinear engagement scenario. The advantage of proposed law will be shown by comparing with PNG derived by using the optimal approach which is

\[
a_M^{PNG} = -N\dot{\lambda} - 3\dot{\lambda}
\]

(32)

And the proposed guidance law is

\[
a_M^{RPG} = -3\dot{\lambda} - Q\text{tanh}(\dot{\lambda})
\]

(33)

The switching control part \( Q\text{tanh}(\dot{\lambda}) \) in the (27) is changed by \( Q\text{tanh}(\dot{\lambda}) \) for alleviating chattering. The constant velocity of the missile and the target are set to be 700 m/s and 500 m/s. The initial flight path angles of the missile and the target are 0 deg. The relative initial distance is 10 km along the \( x \)-axis and 0 m along the \( y \)-axis. Several simulations are conducted with different error source: target acceleration step, ramp and heading error.

Fig. 2 shows the simulation results with the step change of 5 g target acceleration. The target acceleration begins at \( t=3 \) s. The time history of the guidance command of the missile is shown in Fig. 2 (right). At first, PNG and proposed guidance laws show the same histories when there is no target acceleration. After target maneuvering by 5 g, magnitude of the proposed guidance law increases to nullify target acceleration. The maximum acceleration of the PNG is larger than the proposed law. We can see that no chattering occurs.

Fig. 3 shows the simulation results with ramp of 0.25 g/s target acceleration. The time history of the guidance command of the missile is shown in Fig. 3 (right). Similar to the previous results, magnitude of the proposed guidance law increases to nullify target acceleration. The maximum acceleration of the PNG is larger than the proposed law. We can see that no chattering occurs.

Fig. 4 shows the simulation results with the step change of 5 g target acceleration with -20 deg of initial heading error. The target acceleration begins at \( t=3 \) s. The time history of the guidance command of the missile is shown in Fig. 4 (right). The proposed guidance law magnitude is larger than the PNG at first causing by initial heading error. It is also larger than PNG after 3 s to nullify target acceleration. But the maximum acceleration of the PNG is larger than the proposed law. In this strict condition, PNG misses the target when effective navigation constant is 3 while the proposed guidance law can catch the target.

5. CONCLUSION

The robust guidance law based on PNG is developed. First, miss distance is estimated by using linearized engagement dynamics for nonmaneuvering target. Lyapunov approach is used to analyze the miss distance and design the robust guidance when the target acceleration exists. The proposed robust guidance has the form of combination of the conventional PNG and switching part. Moreover, this guidance is simple and apt for implementation in practice. Nonlinear engagement simulation
shows that the proposed guidance law has good performance in intercepting a maneuvering target.

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REFERENCES


