Adaptive Sliding Mode Control for Non-affine Nonlinear Vehicle Systems

Chaeik Ahn*, Youdan Kim†, Hyounjin Kim‡
Seoul National University, Seoul 151-742, Korea

This paper presents an adaptive sliding mode control method for non-affine and non-square nonlinear systems. By differentiating the nonlinear dynamic equation, an increased-order nonlinear system is obtained, and the derivative of the control signal can be used as a new control variable. To overcome the non-square property of the system, the slack variable vector is introduced to make the system input influence matrix square. A systematic procedure is developed and related theoretical and practical issues are discussed. Stability and robustness of the proposed control scheme are also analyzed using Lyapunov approach and LaSalle-Yoshizawa theorem. Numerical simulations performed for a nonlinear UAV and a supercavitation vehicle model demonstrate that the closed-loop system has good performance.

I. Introduction

The most widely used approach to general nonlinear system is the method based on linearization of the nonlinear system model around an operating point. While the linearization may provide sufficiently accurate controllers in the case of tracking desired trajectories, the problem becomes very difficult because the linearization results in a linear but time-varying model. Fixed linear controllers in such a case can result in unacceptable responses. To overcome these problems, nonlinear methods are developed.

Recently, there has been a significant progress in the area of nonlinear control system design. Lewis et al.1 proposed an online neural network that approximated unknown functions to design a controller for a robot. They performed Lyapunov stability analysis that guaranteed both tracking performance as well as the boundedness of weights. A relatively simpler and popular method of nonlinear control design is feedback linearization.2, 3 In this approach, an appropriate coordinate transformation is carried out to make the system dynamics have a linear form. Linear control design tools are then used to synthesize the controller. A drawback of this approach is its sensitivity to modeling errors and parameter uncertainties. Kim and Calise proposed a neural network controller with dynamic inversion so that the inversion error may be cancelled out.4 The feasibility and usefulness of this technique has been demonstrated in many flight control applications. Sliding mode control design has been also applied to nonlinear system control successfully.5, 6 The main advantage of the sliding mode controller is the robust stability of the closed-loop system with respect to the model uncertainties, disturbances, and nonlinear effects due to faults. However, pure sliding mode control has some drawbacks such as large control authority requirements and chattering. To overcome these problems, online adaptive parameter estimation scheme employed in addition to the sliding mode controller.6, 7 The performance of the pure sliding mode controller has been improved by combining it with an on-line parameter estimation schemes. Many of these results as well as adaptive sliding mode control are applicable to the systems that are affine in the control variables. However, many practical applications give rise to non-affine and non-square nonlinear systems. Therefore, there is a need to develop a universal control design technique which can be applied to nonlinear systems in a general form.

One possible approach for input non-affine nonlinear system is to directly invert the nonlinear function of control input. When it is difficult to invert the nonlinearity, the Implicit Function Theorem8 (IMF) can still be used to demonstrate the existence of the corresponding inverse function. However, this approach does not provide a systematic way for determining such an inverse. A method based on the passivity theorem is applicable to systems with Lyapunov stable unforced dynamics, but it cannot deal with the nonlinear systems that do not satisfy the

*Graduate Student, Department of Aerospace Engineering, sety@snu.ac.kr
† Professor, Department of Aerospace Engineering, ydkim@snu.ac.kr, Senior Member AIAA
‡ Professor, Department of Aerospace Engineering, hjinkim@snu.ac.kr, Member AIAA
passivity condition. The other method is based on a well-known idea of differentiating the original state equation, obtaining an augmented model linear in control derivative, and using it as a new control variable. This approach has advantages that it is easy to apply to a variety of nonlinear systems and does not require the existence of the inverse function. Using this method, Boskovic et al. proposed an adaptive controller for non-affine nonlinear aircraft system. However, these methods are only for square systems. The method for both non-affine and non-square system was first presented by Balakrishnan et al. They developed a model following controller by adding an extra control to a nominal controller, for guaranteeing overall stability and performance improvement.

The objective of this paper is to present a new systematic approach of adaptive sliding mode control scheme which can be applied to general non-affine, non-square nonlinear systems. The design procedure relies on the concept of Ref. 10 and Ref. 12. The sliding mode control scheme and online adaptive parameters guarantee the stability robustness of the closed-loop system with respect to the model uncertainties, disturbances and nonlinear effects due to faults. The whole system stability with the augmented adaptive parameters is analyzed using a Lyapunov-based approach. Numerical simulation studies for nonlinear UAV and supercavitation vehicle system problems were carried out to verify the performance of proposed scheme.

II. Control Design Procedure

Consider a nonlinear system,

\[ \dot{X} = F(X, U) + \nu \]  

(1)

where \( X \in \mathbb{R}^n \) is the state vector, \( U \in \mathbb{R}^m \) (\( m \leq n \)) is the control vector of a system, and \( \nu \in \mathbb{R}^n \) is the unknown constant vector which describes the nonlinear effect or modeling error. It is assumed that the order of the system is known, and the vector function \( F(X, U) \) is continuously differentiable. To design a controller for the system of Eq. (1), a few different cases of general nonlinear systems are considered.

A. Case I: Input affine and square system

The system is square (\( m = m \)) and the system dynamics is affine in control variables. Then, Eq. (1) can be rewritten as

\[ \dot{X} = F(X, U) + \nu = f(X) + g(X) \cdot U + \nu \]  

(2)

To design a sliding mode controller, an appropriate sliding surface should be selected in the consideration of the desired dynamic property. Let us define the sliding surface as

\[ S = K_p e + K \int_0^t e \, d\tau \]  

(3)

where \( e = X - X_d \) is the state error, \( K_p \) and \( K \) are gain matrices. The motion of the closed-loop system using the sliding mode control law is composed of two modes. The first mode is a reaching mode where the states beginning from arbitrary initial state are attracted towards the sliding surface. In the second mode, i.e., sliding mode, the states slide along the sliding surface, and the state error converges to zero. In the presence of uncertainty, discontinuous control law is used for accomplishing sliding motion. Once the sliding surface has been chosen, a controller should be designed to make the sliding surface be an attractive surface. Let us differentiate the sliding surface with respect to time as

\[ \dot{S} = K_p (\dot{X} - \dot{X}_d) + K_p e = K_p (f(X) + g(X)U + \nu - \dot{X}_d) + K_p e \]  

(4)

To design the sliding mode controller, matrix \( g(X) \) should be non-singular at any time. If matrix \( g(X) \) is non-singular at any time, then following control input and Lyapunov candidate function can be considered.
where $\tilde{\nu} = \nu - \dot{\nu}$, $\Gamma_\nu$ is a positive definite diagonal matrix, and $c_1$, $c_2$ are positive constants. Differentiating Eq. (6) with respect to time and using Eq. (5) in the resulting equation, we have

$$\dot{\nu} = S^T \left[ K_p \tilde{\nu} - c_1 S - c_2 \text{sgn}(S) \right] + \tilde{\nu}^T \Gamma_\nu^{-1} \tilde{\nu} = -S^T c_1 S - c_2 \|S\| + \tilde{\nu}^T \left( K_p^T S - \Gamma_\nu^{-1} \tilde{\nu} \right)$$

(7)

Note that the relations of $\tilde{\nu} = -\dot{\nu}$ is used to derive the above equation. From Eq. (7), the following adaptation law for $\dot{\nu}$ can be obtained.

$$\dot{\nu} = \Gamma_\nu K_p^T S$$

(8)

Substituting the adaptation law into Eq. (8), we have

$$\dot{\nu} = -S^T c_1 S - c_2 \|S\| \leq 0$$

(9)

From Eq. (9), it can be stated that the equilibrium state is stable. Finally, it can be shown that the error state $e$ tends to zero as $t$ goes to $\infty$ by LaSalle-Yoshizawa theorem.13

B. Case II : Input affine and non-square system

If the system is control affine but non-square, two cases may arise; either $(m > n)$ or $(m < n)$. For the case of $(m > n)$, the control law design problem can be converted into an optimal control allocation problem. In this case, linear or nonlinear programming techniques may be used. For the case of $(m < n)$, which is frequently met in many engineering problems, the number of equations is greater than the number of control variables and Eq. (4) cannot be solved for control inputs. In this study, to solve this problem, extra variables are introduced to augment the control vector to make the system square. From this solution, the components of the augmented control vector representing the actual controller can be extracted.

In order to find the solution of an input-affine and non-square system, a slack-variable vector $U_s$ is introduced first. Next, an $n \times (n - m)$ matrix $\Sigma$ is designed and $\Sigma U_s$ is added and subtracted to the right hand side of Eq. (2). Then, Eq. (2) can be rewritten as

$$\dot{X} = f(X) + G(X) \Lambda + \nu_s$$

(10)

where

$$G \triangleq \left[ g(X) : \Sigma \right]$$

(11)

$$\Lambda \triangleq \left[ U^T : U_s^T \right]^T$$

(12)

$$\nu_s \triangleq \nu - \Sigma U_s$$

(13)

Note that the matrix $\Sigma$ should be chosen carefully such that the square matrix $G(X)$ becomes non-singular. Let us consider the same sliding surface of Eq. (3). Using Eqs. (10)-(13), Eq. (4) can be expressed as
\[ \dot{S} = K_p \left( f(X) + GA + \nu_a - \dot{X}_d \right) + K_e e \]  

(14)

Because the matrix \( G(X) \) is square and nonsingular, the following equation and Lyapunov candidate function can be considered.

\[
\Lambda = \left( K_p G \right)^{-1} \left( -K_p f(X) - K_p \dot{\nu}_a + K_p \dot{X}_d - K_e e - c_1 S - c_2 \text{sgn}(S) \right) 
\]

(15)

\[
L = \frac{1}{2} S^T S + \frac{1}{2} \dot{\nu}_a^T \Gamma_v^{-1} \dot{\nu}_a
\]

(16)

From Eq. (15), an actual control input can be extracted as

\[
U = \Lambda \left( 1: m, \ldots \right)
\]

(17)

Differentiating Eq. (16) with respect to time and using Eq. (15) in the resulting equation, we have

\[
\dot{L} = S^T \left[ K_p \dot{\nu} - c_1 S - c_2 \text{sgn}(S) \right] + \dot{\nu}_a^T \Gamma_v^{-1} \dot{\nu}_a = -S^T c_1 S - c_2 \| S \| + \dot{\nu}_a^T \left( K_p^T S - \Gamma_v^{-1} \dot{\nu}_a \right)
\]

(18)

From Eq. (17), the following adaptation law for \( \dot{\nu}_a \) can be obtained.

\[
\dot{\nu}_a = \Gamma_v K_p^T S
\]

(19)

Substituting the adaptation law into Eq. (18), we have

\[
\dot{L} = -S^T c_1 S - c_2 \| S \| \leq 0
\]

(20)

From Eq. (19), it can be stated that the error state \( e \) tends to zero as \( t \) goes to \( \infty \).

C. Case III: Input non-affine and square system

Consider a system which is square but not control-affine. Since the control input vector does not appear linearly in the system equations, it is difficult to determine the control input directly by inverse. In this study, following the idea in Ref. 10 and Ref. 11, a novel method is introduced to deal with a class of control non-affine smooth nonlinear systems, Eq. (1).

The objective is to find the control input such that the error between the desired system trajectory and the system state vector is minimized. Let us define the Jacobian matrices of the vector function \( F(X,U) \) as follows.

\[
F_x = \frac{\partial F(X,U)}{\partial X}
\]

(21)

\[
F_u = \frac{\partial F(X,U)}{\partial U}
\]

(22)

Let us take the time derivative of Eq. (1) so that \( \dot{U} \) appears linearly in the resulting equation;

\[
\dot{X} = \frac{\partial F(X,U)}{\partial X} \dot{X} + \frac{\partial F(X,U)}{\partial U} \dot{U} = F_x \left[ F(X,U) + \nu \right] + F_u \dot{U}
\]

(23)
To design the sliding mode controller, the following sliding surface is adopted, and the time derivative of sliding surface can be obtained as

\begin{equation}
S = K_d \dot{e} + K_p e + K_i \int_0^t e \, d\tau 
\end{equation}

\begin{equation}
\dot{S} = \left( K_d F_X + K_p \right) F + \left( K_d F_X + K_p \right) \dot{u} + K_d F \dot{X} - K_p \ddot{X} + K_i e 
\end{equation}

If \( F_U \) is nonsingular at any time, the control input derivative and Lyapunov candidate function can be introduced as

\begin{equation}
\dot{U} = \left( K_p F_U \right)^{-1} \left[ -\left( K_d F_X + K_p \right) F - \left( K_d F_X + K_p \right) \dot{u} + \dot{K}_d \ddot{X} + K_p \dot{X} - K_i e - c_i S - c_2 \text{sgn}(S) \right] 
\end{equation}

\begin{equation}
L = \frac{1}{2} S^T S + \frac{1}{2} \dot{\nu} \Gamma_\nu \dot{\nu}
\end{equation}

Differentiating Eq. (27) with respect to time and using Eq. (26) in the resulting equation, we have

\begin{equation}
\dot{L} = S^T \left[ \left( K_d F_X + K_p \right) \dot{U} - c_i S - c_2 \text{sgn}(S) \right] + \dot{\nu} \Gamma_\nu \dot{\nu}
\end{equation}

\begin{equation}
= -S^T c_i S - c_2 \|S\| + \dot{\nu} \left( \left( K_d F_X + K_p \right)^T S - \Gamma_\nu \dot{\nu} \right)
\end{equation}

To make Eq. (28) be negative definite, the following adaptation law for \( \dot{\nu} \) can be used.

\begin{equation}
\dot{\nu} = \Gamma_\nu \left( K_d F_X + K_p \right)^T S 
\end{equation}

Then Eq. (28) becomes

\begin{equation}
\dot{L} = -S^T c_i S - c_2 \|S\| \leq 0
\end{equation}

From Eq. (30), it can be stated that system error dynamics with the control input \( U = \int_0^t \dot{U} \, d\tau \) is stable in the sense of Lyapunov.

**D. Case IV : Input non-affine and non-square system**

Now consider a system which is non-square and non-affine in the control. The system equation can be written as

\begin{equation}
\dot{X} = F_X \left[ F(X, U) + \nu \right] + G(X) \Lambda + \xi 
\end{equation}

where

\begin{equation}
G \triangleq \left[ \begin{array}{c} F_U \\ \Sigma \end{array} \right] 
\end{equation}

\begin{equation}
\Lambda \triangleq \left[ \begin{array}{c} U^T \\ U^T U_S \end{array} \right]^T 
\end{equation}

\begin{equation}
\xi \triangleq -\Sigma U_S 
\end{equation}

Again, like the input affine and non-square case, the matrix \( \Sigma \) should be chosen carefully such that the square matrix \( F_U \) does not become singular. Considering the same sliding surface of Eq. (24) and using the Eq. (31)-(34), the time derivative of sliding surface can be expressed as

5

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\[
\dot{S} = \left( K_d F_x + K_p \right) F + \left( K_d F_x + K_p \right) \dot{u} + K_d G(X) \Lambda + K_p \dot{\xi} - K_d \dot{X}_d - K_p \dot{X}_d + K_e
\]  

(35)

Because the matrix \( G \) is square and nonsingular, the following equation and Lyapunov candidate function can be considered.

\[
\Lambda = \left( K_d G \right)^{-1} \left\{ -\left( K_d F_x + K_p \right) F - \left( K_d F_x + K_p \right) \dot{u} - K_d \dot{\xi} + K_d \dot{X}_d + K_p \dot{X}_d - K_e - c_1 S - c_2 \text{sgn}(S) \right\}
\]  

(36)

\[
L = \frac{1}{2} S^T S + \frac{1}{2} \dot{\xi}^T \Gamma^{-1} \dot{\xi} + \frac{1}{2} \xi^T \Gamma^{-1} \xi
\]  

(37)

From Eq. (36), the actual control input can be extracted. Differentiating Eq. (37) with respect to time and using Eq. (36) in the resulting equation, we have

\[
\dot{L} = S^T \left[ \left( K_d F_x + K_p \right) \dot{u} + K_d \dot{\xi} - c_1 S - c_2 \text{sgn}(S) \right] + \dot{\xi}^T \Gamma^{-1} \dot{\xi} + \xi^T \Gamma^{-1} \xi
\]

\[
- = -S^T c_1 S - c_2 \|S\|^2 + \dot{\xi}^T \left( K_d F_x + K_p \right)^T S - \Gamma^{-1} \dot{\xi} + \xi^T \left( K_d S - \Gamma^{-1} \xi \right)
\]  

(38)

From Eq. (38), the adaptation law for \( \dot{\xi} \) and \( \dot{\xi} \) can be obtained as

\[
\dot{\xi} = \Gamma \dot{u}
\]  

(39)

\[
\dot{\xi} = \Gamma \dot{\xi}
\]  

(40)

Substituting the adaptation law into Eq. (38), we have

\[
\dot{L} = -S^T c_1 S - c_2 \|S\|^2 \leq 0
\]  

(41)

From Eq. (20), it can be stated that the error state \( e \) tends to zero as \( t \) goes to \( \infty \).

### III. Numerical Simulation Studies

In this section, two numerical examples that demonstrate the performance of the proposed control law are presented. The examples show that the control law proposed in this paper can be used to design control system for complex nonlinear systems.

Note that the control law in Eqs. (5), (15), (26) and (36) include discontinuous functions which can cause chattering. Chattering is undesirable because it involves very high control action and may excite high frequency unmodeled dynamics. The discontinuity in the control law can be dealt with by adopting a thin boundary layer around the sliding surfaces, that is, replacing \( \text{sgn}(S) \) with the continuous saturation functions \( \text{sat}(S/\epsilon) \), where \( \text{sat}(\eta) = \eta \) if \( |\eta| \leq 1 \), and \( \text{sat}(\eta) = \text{sgn}(\eta) \) otherwise.

#### A. Input non-affine and square system : Nonlinear UAV model

As the first example, nonlinear non-affine 3 DOF UAV model is considered.\(^{14}\) The differential equations of the point-mass UAV dynamics are given by
\[
\dot{\gamma} = g \left( \frac{T - D}{W} - \sin \gamma \right)
\]
\[
\dot{\chi} = \frac{g n \sin \mu}{V \cos \gamma}
\]

Flight trajectory can be generated by the following equations.

\[
\begin{align*}
\dot{x} &= V \cos \gamma \cos \chi \\
\dot{y} &= V \cos \gamma \sin \chi \\
\dot{h} &= V \sin \gamma
\end{align*}
\]  

The state variables are airspeed \( V \), flight path angle \( \gamma \), flight path heading angle \( \chi \), and the control variables are thrust \( T \), load factor \( n \), and bank angle \( \mu \). UAV position variables \( (x, y, h) \) are represented in the inertial frame. This is a square \( (n = m = 3) \) but input non-affine problem. The drag force \( D \) is represented by a simple drag polar model as

\[
D = \frac{1}{2} \rho V^2 SC_{Dh} + \frac{2kn^2W^2}{\rho V^2 S}
\]  

Detailed UAV model parameters are summarized in Table 1.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight, ( W )</td>
<td>14,515 kg</td>
</tr>
<tr>
<td>Reference area, ( S )</td>
<td>37.16 m²</td>
</tr>
<tr>
<td>Maximum thrust, ( T )</td>
<td>113,800 N</td>
</tr>
<tr>
<td>Maximum lift coefficient, ( C_{l_{\text{max}}} )</td>
<td>2.0</td>
</tr>
<tr>
<td>Maximum load factor, ( n_{\text{max}} )</td>
<td>7</td>
</tr>
<tr>
<td>Induced drag coefficient, ( k )</td>
<td>0.1</td>
</tr>
<tr>
<td>Parasite drag coefficient, ( C_{D_{\text{p}}} )</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The initial flight condition is a level flight with \( V = 91.44 \text{ m/s} \) at \( h = 3048 \text{ m} \). To generate a differentiable command signal, reference command is transferred to the controller through the following command filter.

\[
\frac{x_{d}(s)}{x_{c}(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]  

where \( \omega_n = 3 \text{ rad/s} \), and \( \zeta = 1 \). In all simulations, it is assumed that the design objective is to assure that the forward velocity \( V \) and flight path angle \( \gamma \) are regulated around the desired values \( [300 \quad 0]^T \), while the heading angle \( \chi \) follows an 30° heading angle command as follows.\(^{11}\)
The control gain and design parameters are selected as follows.

\[
\begin{align*}
    c_1 &= 2 \cdot I_1, \quad c_2 = 2 \cdot I_1 \\
    K_d &= 0.5 \cdot I_3, \quad K_p = 0.5 \cdot I_3, \quad K_i = 0 \\
    \Gamma_u = \Gamma_u = I_1, \quad \epsilon = 0.1
\end{align*}
\]

(47)

Fig. 1 shows the resulting state trajectories of the system with the adaptive sliding mode controller and the reference command signals. Figs. 2 and 3 illustrate the control trajectories and the estimated value of the on-line updated unknown parameter, respectively. The response is acceptable and the control objective is achieved with all control inputs within the saturation bounds.
Fig. 2 Control histories (UAV)

Fig. 3 On-line updated parameter vector (UAV)
B. Input non-affine and non-square system: Supercavitation vehicle model

Cavitation is defined as the formation of bubbles in a liquid subject to local pressure variations. Cavitation must be avoided in most of engineering applications, but supercavitation vehicle uses this undesirable phenomenon. A high speed supercavitation vehicle with a cavitator can generate a cavity bubble covering nearly the entire body. Most of the supercavitation vehicle surface is not wetted, which leads to significant drag reduction and enables high speed under the water. However, there is very low buoyancy acting on the body, because most of the body is not wetted. The vehicle weight must be supported by the lift from the cavitator and the fins. The body can also touch the cavity wall which leads to very high restoring force, however it is not desirable in most situations because of the limit cycle induced by it.

Various efforts have been carried out to model the dynamics of a supercavitating vehicle. Dzielski and Kurdila\textsuperscript{15} proposed a simplified dive-plane model with an assumed analytical form for the fin forces. The vehicle can be controlled with the deflections of cavitator and fins. In this paper, supercavitation vehicle is modeled using the information in Ref. 15. The supercavitation vehicle is modeled as an input non-affine and non-square nonlinear system.

1. Modeling of longitudinal dynamics

In this study, a longitudinal dynamics of a rigid body vehicle is considered. A local body-fixed frame and inertial frame are shown in Figure 4.

\[
\begin{align*}
\begin{bmatrix}
m & -mX_g & -mX_g \\
-mX_g & I_{yy} & -mX_g \\
\end{bmatrix}
\begin{bmatrix}
\dot{w} \\
\dot{\theta}_p \\
\end{bmatrix}
= & \begin{bmatrix}
m \cdot u \cdot q \\
-m \cdot X_g \cdot u \cdot q \\
\end{bmatrix}
+ \begin{bmatrix}
F_{cont} + F_{grav} + F_{plane} \\
M_{cont} + M_{grav} + M_{plane} \\
\end{bmatrix}
\end{align*}
\]

(48)

where \( m \) is the total mass, \( I_{yy} \) is the moment of inertia, \( X_g \) is the location of the center of gravity relative to the cavitator, \( q \) is the pitch rate, \( u \) and \( w \) are the linear velocity in the \( x \) and \( z \)-axis, respectively. The first term in the right hand side due to the fact that the coordinate origin for the dynamics is not the center of gravity. Subscripts \( cont \) means contribution from control surfaces, \( grav \) and \( plane \) means contribution from gravity and planning force, respectively.

If the full vehicle body is in the cavity, hydrodynamic forces will only act on the cavitator and the fins. The cavitator drag coefficient is modeled as \( C_d = C_{d0} (1 + \sigma) \), where \( C_{d0} = 0.82 \). The resulting lift on the cavitator, the total vertical force acting on control surfaces, and the moment due to control forces can be modeled as follows.
\[ F_{\text{cont}} = F_{\text{cov}} + F_{\text{fin}} = C_i \sin \left( \frac{w + \delta}{u} \right) \sin \left( \frac{w + qL}{u} + \delta_f \right), \quad \text{where} \quad C_i = \frac{1}{2} \pi \rho R^2 V^2 C_s \]

\[ M_{\text{cont}} = L \cdot F_{\text{fin}} = -nLC_i \sin \left( \frac{w + qL}{u} + \delta_f \right) \]

where \( \rho \) is the density of the water, \( R_s \) is the radius of the cavitation, \( V \) is the stream velocity, \( L \) is the vehicle length, \( n \) is the effectiveness of the control surfaces in provided lift as a function of angle of attack relative to the effectiveness of cavitation, \( \delta_c \) and \( \delta_f \) are cavitation and fin deflection angles, respectively. It is assumed that the fins are located at the back of the vehicle and have a moment arm \( L \) relative to the reference point. The difference between the previous work\textsuperscript{15, 16} and this study is that the small angle assumption is not used, and the obtained equations are input non-affine form. The force and moment due to gravity can be simply expressed as

\[
\begin{bmatrix}
  F_{\text{grav}} \\
  M_{\text{grav}}
\end{bmatrix} =
\begin{bmatrix}
  mg \\
  -X_g \cdot mg
\end{bmatrix}
\]  

(50)

The last terms in the equation (48) represents the force and moment due to the planning of the body on the cavity. According to the Logvinovich’s analysis, they can be described as follows\textsuperscript{15}.

\[
\begin{bmatrix}
  F_{\text{plane}} \\
  M_{\text{plane}}
\end{bmatrix} =
\begin{bmatrix}
  \pi \rho R^2 V^2 \\
  -\pi \rho R^2 V^2 \cdot L
\end{bmatrix}
\left[
\begin{bmatrix}
  \pi \rho R^2 V^2 \\
  -\pi \rho R^2 V^2 \cdot L
\end{bmatrix}
\right]
\left[
\begin{bmatrix}
  1 - \left( \frac{R'}{h'+R'} \right)^2 \\
  \frac{1}{1+2h'}
\end{bmatrix}
\right]
\alpha_p
\]  

(51)

The normalized difference between the cavity and the body diameter \( R' \), the immersion depth \( h' \), and the angle of attack \( \alpha_p \) used in determining the planning force are given by

\[
R' = \frac{R - R}{R} \]

(52)

\[
h' = \begin{cases} 
  0 & , \quad R' > \frac{L|w|}{RV} \\
  \frac{L|w|}{RV} - R', & \quad \text{otherwise}
\end{cases} \]

(53)

\[
\alpha_p = \begin{cases} 
  \frac{w - \hat{R}}{V} & , \quad \frac{w}{V} > 0 \\
  \frac{w + \hat{R}}{V}, & \quad \text{otherwise}
\end{cases}
\]

(54)

where \( \hat{R} < 0 \) is the cavity radius contraction rate. The planning force with respect to vertical speed (\( w \)) is shown in Figure 5. From the profile of the planning force with respect to the vertical speed, it can be seen that planning force is a kind of restoring. Additional details regarding this model can be found in Ref. 15.
Fig. 5 Normalized planning force vs. the vertical speed\textsuperscript{16}

The state space description of the vehicle is extended by adding two more state variables, the position of the vehicle’s nose along the \(z\)-axis and the vehicle pitch angle. Finally, the extended state space model can be expressed as

\[
\begin{bmatrix}
\dot{z} \\
\dot{\theta} \\
\dot{w} \\
\dot{q}
\end{bmatrix} = \begin{bmatrix}
w - V \sin \theta \\
q \\
M^{-1} \cdot N(X,U)
\end{bmatrix} + \begin{bmatrix}
0_{2\times 1} \\
\begin{bmatrix} F_{\text{plane}} \\ L \cdot F_{\text{plane}} \end{bmatrix}
\end{bmatrix} \triangleq F(X,U) + \nu
\]

(55)

where matrices \(M\) and \(N(X,U)\) are defined as

\[
M = \begin{bmatrix}
m & -m X_G \\
-m X_G & I_{yy}
\end{bmatrix}
\]

(56)

\[
N(X,U) = \begin{bmatrix}
m u q + C_i \sin \left( \frac{w}{u} + \delta_i \right) - n C_i \sin \left( \frac{w + q L}{u} + \delta_f \right) + mg \\
-m X_G u q - n L C_i \sin \left( \frac{w + q L}{u} + \delta_f \right) - X_G \cdot mg
\end{bmatrix}
\]

(57)

This nonlinear model is non-square (\(n = 4\), \(m = 2\)) and non-affine in the control variables. The parameter values of Eqs. (55) and (56) are summarized in Table 2.
Table 2 Vehicle parameters\textsuperscript{15}

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravitational acceleration, $g$</td>
<td>$9.81\text{ m/s}^2$</td>
</tr>
<tr>
<td>Water density, $\rho$</td>
<td>$1000\text{ kg/m}^3$</td>
</tr>
<tr>
<td>Uniform density for the vehicle body, $\rho_b$</td>
<td>$2 \cdot \rho$</td>
</tr>
<tr>
<td>Vehicle length, $L$</td>
<td>$1.80\text{ m}$</td>
</tr>
<tr>
<td>Vehicle radius, $R$</td>
<td>$0.0508\text{ m}$</td>
</tr>
<tr>
<td>Cavitator radius, $R_c$</td>
<td>$0.0191\text{ m}$</td>
</tr>
<tr>
<td>Velocity, $V$</td>
<td>$75\text{ m/s}$</td>
</tr>
<tr>
<td>Cavitation number, $\sigma$</td>
<td>$0.03$</td>
</tr>
<tr>
<td>Drag coefficient, $C_{\Delta}$</td>
<td>$0.82$</td>
</tr>
<tr>
<td>Fin effectiveness, $n$</td>
<td>$0.5$</td>
</tr>
<tr>
<td>Vehicle mass, $m$</td>
<td>$7(\rho_b \pi)R^2L/9$</td>
</tr>
<tr>
<td>Moment of inertia, $I_{yy}$</td>
<td>$11(\rho, \pi)R^4L/60 + (\rho, \pi)R^2L^3$</td>
</tr>
<tr>
<td>Location of the center of gravity, $X_G$</td>
<td>$-17L/28$</td>
</tr>
</tbody>
</table>

2. Simulation results of the supercavitation vehicle.

Numerical simulation is performed to verify the performance of the proposed control law. The reference trajectory is an obstacle avoidance maneuver.\textsuperscript{17} We assumed that the vehicle speed $V$ is constant at $75\text{ m/s}$ while the vehicle moves downward $10\text{ m}$ and returns to continue its straight path. The additional reference signals are derived using further assumption to generate the physically feasible reference trajectories. The vertical position change is caused by the vertical speed and the projection of the longitudinal velocity to the vertical plane. Previous results suggested that the vertical speed is closely related to the planning force, and therefore it is desired to be kept as small as possible. Therefore, after designing the reference trajectory for the depth, additional reference signals can be calculated from the following equations.

$$\theta_d = -\frac{\dot{z}_d}{V}, \quad w_d = 0, \quad q_d = \dot{\theta}_d$$ \hspace{1cm} (58)

Note that the reference signals are transferred to the controller through an appropriate command filter to generate the differentiable command signal. The controller does not use the information about the planning force which the vehicle is actually subject to, during its operation, thus the planning force terms in the equations of motion can be treated as unknown vector $\nu$. The control gain and design parameters are chosen as follows.

$$c_1 = 5 \cdot I_3, \quad c_2 = 5 \cdot I_3, \quad K_d = 5 \cdot I_3, \quad K_p = 3 \cdot I_3, \quad K_i = 0$$

$$\Gamma_o = 0.01 \cdot I_3, \quad \Gamma_i = 2 \cdot I_3, \quad \epsilon = 0.1$$ \hspace{1cm} (59)

$$\Sigma = \begin{bmatrix} -10 & 0 & 10 & 0 \\ 10 & -10 & 0 & 10 \end{bmatrix}$$

Numerical results are illustrated in Figs. 6-9. The state trajectories are shown in Fig. 6. It can be seen that the adaptive sliding mode controller does a better job of forcing the state variables to track the reference signals. Figs. 7-9 illustrate the control trajectories and the estimated value of on-line updated unknown parameter. It is seen that the response is acceptable and that the control objective is achieved within the saturation bounds and rate limits.
Fig. 6 State trajectories (Supercavitation Vehicle)

Fig. 7 Control histories (Supercavitation Vehicle)
Fig. 8 On-line updated parameter $\nu$ (Supercavitation Vehicle)

Fig. 9 On-line updated parameter $\xi$ (Supercavitation Vehicle)
IV. Conclusion

An adaptive sliding mode control law with on-line updated unknown parameters has been developed for a fairly general class of nonlinear systems which may be non-square and non-affine in the control variables. The nonlinear system for which the method is applicable is assumed to be of the known order, however it may contain unmodeled dynamics and/or parameter uncertainty. The stability of the proposed control law with adaptation rule was proved by Lyapunov theory. Numerical simulations have been performed for two nonlinear systems. The performance of the proposed technique has been demonstrated by applying it to a non-affine and square UAV system and to non-affine and non-square supercavitation vehicle. The simulation results show that the proposed adaptive sliding mode controller achieves good tracking performance with on-line parameter adaptation.

Acknowledgments

This research was performed for Smart UAV Development Program, one of the 21 Century Frontier R&D Programs funded by the Ministry of Commerce, Industry and Energy of Korea.
References